Lesson Objectives

By the end of this lesson, you should be able to:

- Identify and classify different types of numbers.
- Differentiate between rational and irrational numbers.
- Provide examples of each type of number.
- Apply numbers to real-life situations.

Types of Numbers

Numbers are the foundation of mathematics, used to count, measure, and label objects. They help us understand quantities, perform calculations, and solve real-world problems. From simple counting to complex mathematical models, numbers play a vital role in daily life, science, engineering, and technology. In this lesson, we explore different types of numbers and their unique properties.

Natural Numbers ($\mathbb{N}$)

Definition: Natural numbers are the counting numbers, excluding zero. Counting numbers (Natural numbers) starting from 1: {1, 2, 3, 4, . . . }. It is denoted as $\mathbb{N}$.

$$\mathbb{N} = \{1, 2, 3, 4, \dots\}$$

Examples:

- The number of students in a competition.
- Counting objects like counting the number of chairs in a room.

Quick Question: Is 0 a natural number?

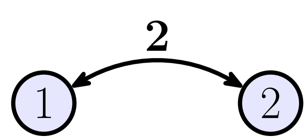
Prime and Composite Numbers

Prime Numbers

A prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself. That is, any prime number only have two factors, itself and 1.

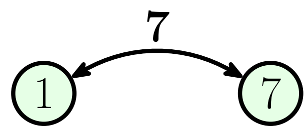
Examples of Prime Numbers:

- 2 is a prime number because it has only two factors: 1 and 2.
- 3, 5, 7, 11, and 13 are also prime because they each have exactly two factors.
- **Example: 2** is a prime number. Its factors are 1 and 2.



$$1 \times 2 = 2$$

- **Example:** **7** is a prime number. Its factors are 1 and 7.



$$1 \times 7 = 7$$

How to Find Prime Numbers?

To determine whether a number is prime, we check if it has exactly two distinct positive divisors: 1 and itself. One efficient method is to use **divisibility tests** to eliminate numbers that are divisible by smaller prime numbers.

If a number is divisible by any smaller prime number (like 2, 3, 5, or 7), then it is not a prime number. These tests are useful in identifying composite numbers quickly.

Divisibility Rules (Useful for Prime Number Identification)

Divisible by	Rule
2	If the number ends in 0, 2, 4, 6, or 8.
3	If the sum of the digits is divisible by 3.
4	If the number formed by the last two digits is divisible by 4.
5	If the number ends in 0 or 5.
6	If the number is divisible by both 2 and 3.
7	Double the last digit, subtract it from the rest of the number. If the result is divisible by 7, then so is the original number. Repeat if necessary.
9	If the sum of the digits is divisible by 9.
10	If the number ends in 0.

Example: Is 29 prime?

- The square root of 29 is approximately 5.38.
- Prime numbers less than or equal to 5 are: 2, 3, 5.
- 29 is not divisible by 2, 3, or 5.

Conclusion: 29 is a prime number.

Composite Numbers

A **composite number** is a natural number greater than 1 that is *not* a prime number. In other words, it can be formed by multiplying two or more smaller natural numbers. Equivalently, a composite number has more than two positive divisors.

Key Properties of Composite Numbers:

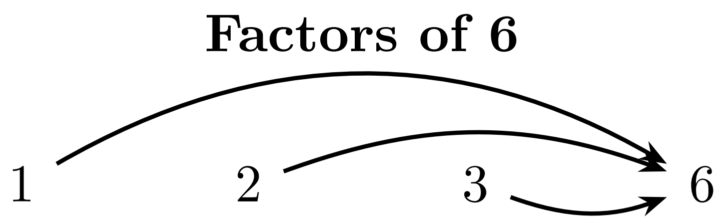
- Every composite number has at least one positive divisor other than 1 and itself.
- The smallest composite number is 4.
- All even numbers greater than 2 are composite because they are divisible by 2.

Definition (Formal): A number $n \in \mathbb{N}, n > 1$ is composite if there exist integers $a, b \in \mathbb{N}$ such that:

$$1 < a < n, \quad 1 < b < n, \quad \text{and} \quad a \times b = n.$$

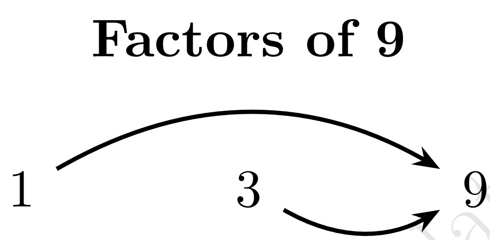
Examples with Visual Diagrams

- **Example: 6 is a composite number.** It has the factors 1, 2, 3, and 6.



$$1 \times 6 = 6, \quad 2 \times 3 = 6$$

- **Example: 9 is a composite number.** Its factors are 1, 3, and 9.



$$1 \times 9 = 9, \quad 3 \times 3 = 9$$

Quick Check: Prime or Composite?

Use divisibility rules to test whether a number is divisible by any smaller number (e.g., 2, 3, 5, 7). If it is, then it's a composite number.

Example: Is 91 a composite number?

- 91 is not divisible by 2, 3, or 5.
- Try 7: $91 \div 7 = 13 \Rightarrow 91 = 7 \times 13$

Conclusion: 91 is a composite number.

Example of Divisibility by 7

Let us test whether 203 is divisible by 7:

- Take the last digit: 3
- Double it: $3 \times 2 = 6$
- Subtract from the remaining digits: $20 - 6 = 14$
- Since 14 is divisible by 7, **203 is divisible by 7**

Hence, 203 is a composite number.

Table: Classification of Numbers from 1 to 20

Number	**Classification**	**Factors**
1	Neither Prime nor Composite	1
2	Prime	1, 2
3	Prime	1, 3
4	Composite	1, 2, 4
5	Prime	1, 5
6	Composite	1, 2, 3, 6
7	Prime	1, 7
8	Composite	1, 2, 4, 8
9	Composite	1, 3, 9
10	Composite	1, 2, 5, 10
11	Prime	1, 11
12	Composite	1, 2, 3, 4, 6, 12
13	Prime	1, 13
14	Composite	1, 2, 7, 14
15	Composite	1, 3, 5, 15
16	Composite	1, 2, 4, 8, 16
17	Prime	1, 17
18	Composite	1, 2, 3, 6, 9, 18
19	Prime	1, 19
20	Composite	1, 2, 4, 5, 10, 20

Whole Numbers ($\mathbb{W}$)

Whole numbers are the set of non-negative integers. They include:

$$\mathbb{W} = \{0, 1, 2, 3, \dots\}$$

Whole numbers cannot be negative.

Examples:

- The number of apples in a basket.
- The count of students in a class.
- The number of pages in a book.

Integers ($\mathbb{Z}$)

Integers are the set of all positive and negative whole numbers, including zero:

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

Integers can be categorized into three parts as shown below:

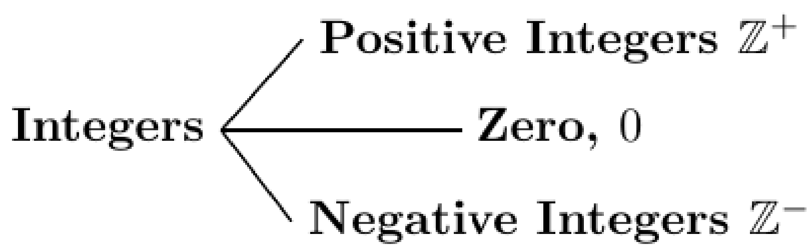


Figure 1: Classification of Numbers

Classification of Integers

- **Positive Integers** ($\mathbb{Z}^+$):
$$\mathbb{Z}^+ = \{1, 2, 3, 4, \dots\}$$
- **Zero** (0)
- **Negative Integers** ($\mathbb{Z}^-$):
$$\mathbb{Z}^- = \{\dots, -4, -3, -2, -1\}$$

Key Insights:

- Every whole number is an integer, but not every integer is a whole number.
- Integers cannot be fractions or decimals.

Examples:

- A temperature of -5°C.
- A bank transaction of +100 Naira.
- Elevation above or below sea level.

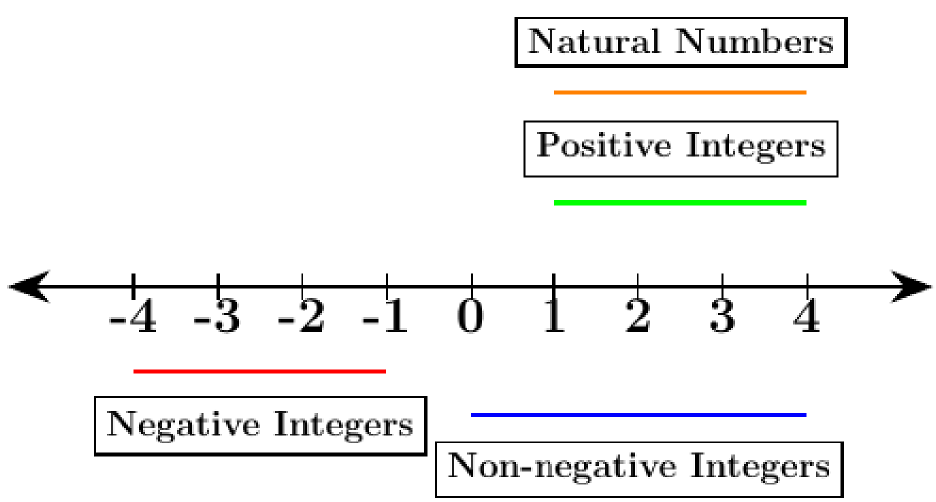


Figure 2: Classification of Numbers

In summary,

$$\mathbb{N} \subset \mathbb{W}$$

and

$$\mathbb{W} \subset \mathbb{Z}$$

Here $\subset$ means "is subset of".

Even and Odd Numbers: All You Must Know

Definition

- **Even numbers** are integers divisible by 2. That is, an integer n is even if there exists an integer k such that $n = 2k$.
- **Odd numbers** are integers not divisible by 2. That is, an integer n is odd if there exists an integer k such that $n = 2k + 1$ or $n = 2k - 1$.

Examples

- **Even Numbers:** $\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots$
- **Odd Numbers:** $\ldots, -5, -3, -1, 1, 3, 5, 7, \ldots$

Important Notes

- Even and odd numbers are **integers**, and hence they include negative values as well.
- **Zero is an even number** because $0 \div 2 = 0$, which is an integer.
- The set of even and odd numbers is infinite in both positive and negative directions.

Classification Table

Number	**Classification**
-5	Odd
-4	Even
-3	Odd
-2	Even
-1	Odd
0	Even
1	Odd
2	Even
3	Odd
4	Even

Application

Understanding even and odd numbers is foundational in:

- Divisibility rules
- Modular arithmetic
- Number theory
- Programming logic and algorithms

1 Rational Numbers ($\mathbb{Q}$)

Definition: Numbers that can be written as fractions $\frac{p}{q}$ where $q \neq 0$.

Examples:

- $\frac{1}{2}$, -$\frac{3}{4}$, 5, 0.75, -2.6
- A recipe requiring $\frac{1}{2}$ teaspoon of salt.
- A discount of 25

Irrational Numbers

Definition: These are numbers that cannot be written as a fraction of two integers. Their decimal representations are non-terminating and non-repeating.
Examples: $\pi, \sqrt{2}, \sqrt{3}, e$

Curious Check!

Try this: Check $\sqrt{2}$ in your calculator. What do you notice?

- Yes! It is approximately 1.414...
- Can you write this as a simple fraction? **No!**
- The decimal goes on and on *without repeating*.
- $\pi = 3.1415926535\ldots$ (Ratio of circumference to diameter of a circle)
- $\sqrt{2} = 1.414213\ldots$ (Square root of 2)
- $e = 2.718281828\ldots$ (Euler's number, fundamental in calculus)

Summary - Fun Fact!

Did You Know?

- **Integers** cannot be fractions or decimal numbers.
- **Irrational Numbers** cannot be expressed as fractions.
- Their decimal parts are *neither terminating nor repeating*.
- The **square root of all prime numbers** is irrational.
- This means $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \sqrt{11}, \ldots$ are all irrational.
- They cannot be written as fractions.
- Their decimal parts go on forever.

These numbers are crucial in geometry, algebra, and physics.

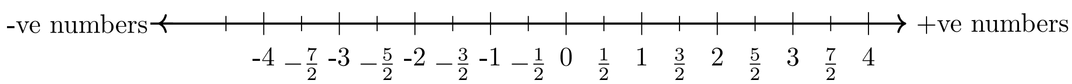
2 Real Numbers ($\mathbb{R}$)

Definition: The set of **real numbers** ($\mathbb{R}$) consists of all numbers that can be represented on the number line. This includes both rational and irrational numbers. That is, real numbers can natural, integers, whole number, decimals, rationals(fractions), irrational numbers

2.1 The Real Number Line

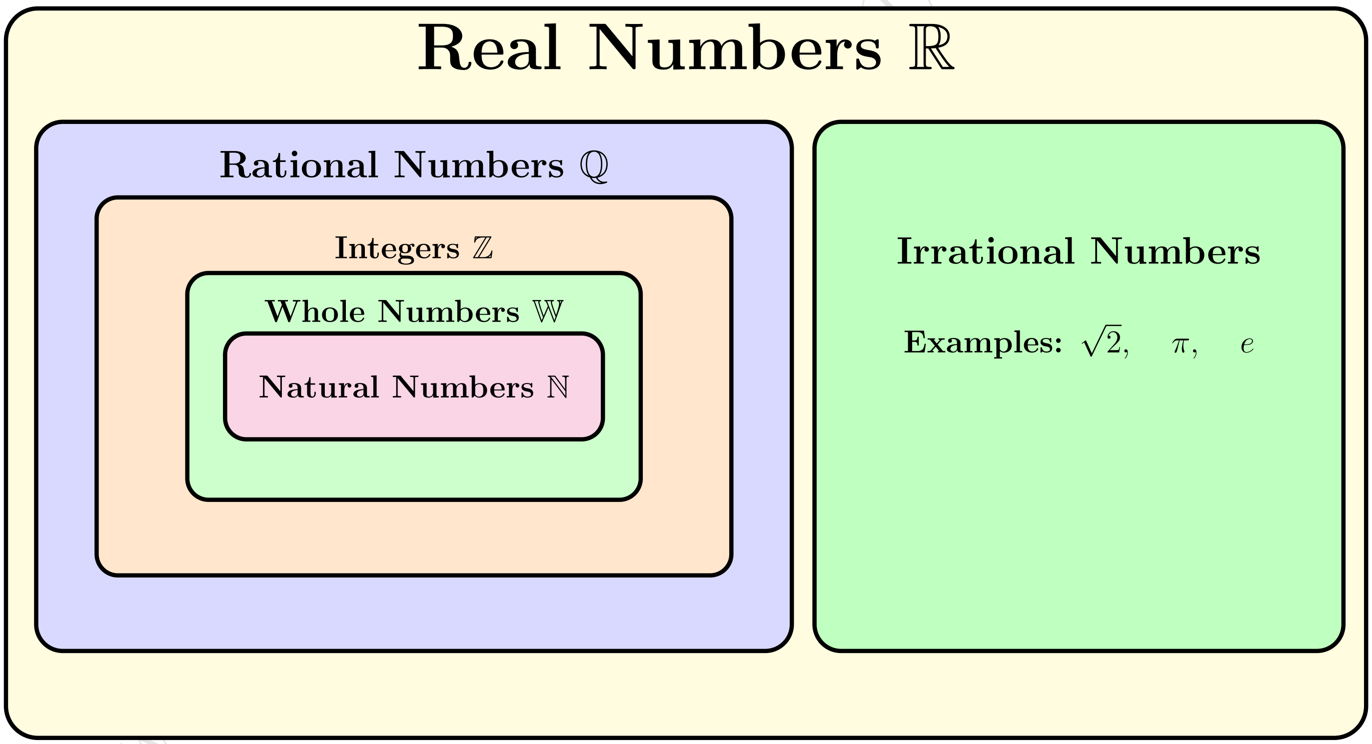
Real numbers can be represented on a continuous number line, where:

- Negative numbers extend to the left.
- Positive numbers extend to the right.
- Zero (0) is at the center, acting as the boundary between positive and negative numbers.



Conclusion: The real number line extends infinitely in both directions, containing all rational and irrational numbers. In summary, real numbers is the combination of rational and irrational numbers.

$$\mathbb{R} = \mathbb{Q} \cup \mathbb{I}$$



3 Complex Numbers ($\mathbb{C}$)

To understand complex numbers, we introduce the imaginary number, denoted as i.

Consider $\sqrt{-25}$. Inputting this into a calculator results in a "Math Error" because the square root of a negative number is undefined in the real number system.

We rewrite:

$$\sqrt{-25} = \sqrt{-1 \times 25} = \sqrt{-1} \times \sqrt{25} = 5i$$

where i is the **imaginary unit** defined as:

$$i = \sqrt{-1}$$

3.1 Properties of the Imaginary Unit

$$i^2 = -1$$
$$i^3 = i^2 \times i = (-1) \times i = -i$$
$$i^4 = i^2 \times i^2 = (-1) \times (-1) = 1$$
$$i^5 = i^4 \times i = 1 \times i = i$$

The powers of i follow a cycle:

$$i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1$$

3.2 Definition of Complex Numbers

A complex number is defined as:

$$z = a + bi, \quad \text{where } a, b \in \mathbb{R}$$

- a is the **real part**, denoted as $\mathrm{Re}(z) = a$.
- b is the **imaginary part**, denoted as $\mathrm{Im}(z) = b$.
- If $b = 0$, the complex number is a real number.
- If $a = 0$, the number is purely imaginary.

3.3 Operations with Complex Numbers

Addition and Subtraction:

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

Multiplication:

$$(a + bi)(c + di) = ac + adi + bci + bdi^2$$

Using $i^2 = -1$:

$$(a + bi)(c + di) = ac - bd + (ad + bc)i$$

Division: To divide two complex numbers, we multiply both numerator and denominator by the conjugate of the denominator.

Given:

$$\frac{a + bi}{c + di}$$

Multiply by $\frac{c-di}{c-di}$:

$$\frac{(a + bi)(c - di)}{(c + di)(c - di)}$$

Using $c^2 + d^2$ in the denominator, this simplifies to:

$$\frac{(ac + bd) + (bc - ad)i}{c^2 + d^2}$$

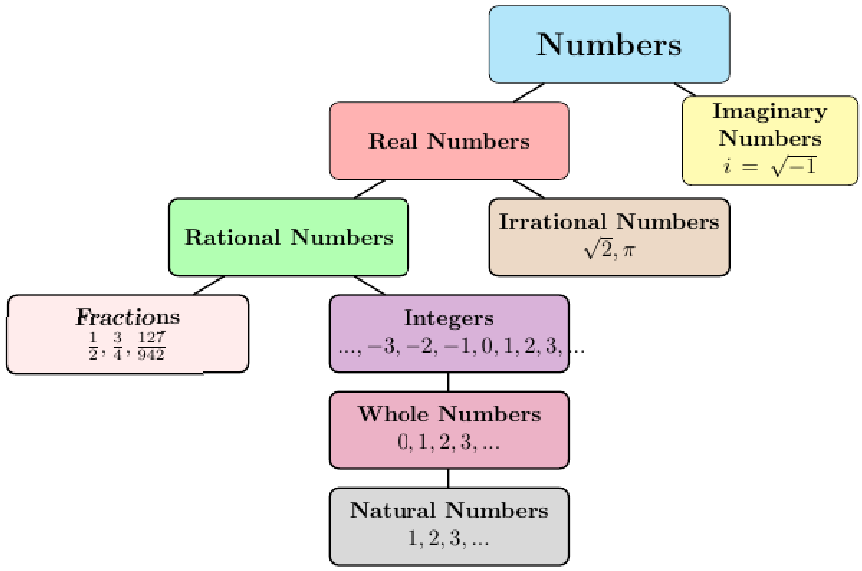


Figure 3: Classification of Numbers

3.4 Summary

- $\mathbb{N}$ (Natural numbers): $\{1, 2, 3, \dots\}$ (Does not include 0)
- $\mathbb{W}$ (Whole numbers): $0, 1, 2, 3, 4, \dots$
- $\mathbb{Z}$ (Integers): $\{\dots, -2, -1, 0, 1, 2, \dots\}$ (Does not include fractions and decimal numbers)
- $\mathbb{Q}$ (Rational numbers): Numbers that can be written as $\frac{a}{b}$, where $a, b \in \mathbb{Z}$ and $b \neq 0$
- $\mathbb{R}$ (Real numbers): Includes all rational and irrational numbers
- $\mathbb{C}$ (Complex numbers): Includes all real and imaginary numbers

$$\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

This means, all the elements of natural numbers are found in whole numbers, whole numbers are found in integers, integers are found in rational numbers, rational and irrational numbers are found in real numbers, and real numbers are part of the complex numbers.

4 Exercises

1. Identify whether the following numbers are Whole, Integer, Rational, Irrational, or Real:

   (a) 3
   (b) -5
   (c) $\frac{7}{2}$
   (d) $\sqrt{5}$
   (e) π

2. Convert the following fractions into decimal form and classify them:

(a) $\frac{1}{3}$
(b) $\frac{5}{8}$

3. Represent the following numbers on a number line:

   (a) -2
   (b) 3.5
   (c) $-\frac{3}{4}$

4. Solve for x:

$$x^2 + 4 = 0$$

5 Interactive Activity

1. Identify the type of each number from the list below and explain your choice.

   (a) -5
   (b) $\frac{7}{3}$
   (c) 4.56
   (d) π
   (e) 12
   (f) $-\sqrt{3}$

2. **Test Your Understanding**
   Which of the following is an integer?

   (a) 3.5
   (b) -2
   (c) $\frac{5}{2}$
   (d) π

3. **True or False**
   State whether the following statements are **true** or **false**.

   (a) Every integer is a whole number.
   (b) Every rational number is a real number.
   (c) An irrational number has a repeating decimal pattern.
   (d) 0 is a natural number.
   (e) $\frac{22}{7}$ is a rational number.

4. **Short Answer Question**
   Explain why $\sqrt{4}$ is a rational number but $\sqrt{2}$ is not.

Answers

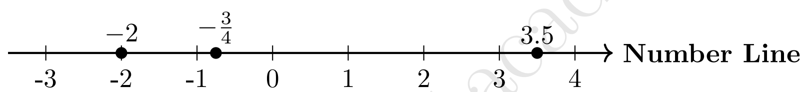
1. **Solution:**

   (a) 3 - Whole, Integer, Rational, Real
   (b) -5 - Integer, Rational, Real
   (c) $\frac{7}{2}$ - Rational, Real
   (d) $\sqrt{5}$ - Irrational, Real
   (e) π - Irrational, Real

   (a) $\frac{1}{3} = 0.3333\ldots$ - Rational (Repeating Decimal)
   (b) $\frac{5}{8} = 0.625$ - Rational (Terminating Decimal)

2. Representation on a number line:

   (a) -2 - Plotted at two units to the left of zero
   (b) 3.5 - Plotted between 3 and 4
   (c) $-\frac{3}{4}$ - Plotted between -1 and 0



3. Solving for x:

$$x^2 + 4 = 0$$

$$x^2 = -4$$

   Since there is no real number whose square is negative, the solutions are imaginary:

$$x = \pm 2i$$

6 Interactive Activity - Solutions

1. Classification of numbers:

   (a) -5 - Integer, Rational, Real
   (b) $\frac{7}{3}$ - Rational, Real
   (c) 4.56 - Rational, Real
   (d) π - Irrational, Real
   (e) 12 - Whole, Integer, Rational, Real
   (f) $-\sqrt{3}$ - Irrational, Real

2. The integer among the given options is:

$$\mathbf{-2}$$

3. True or False:

(a) **False** - Negative integers are not whole numbers.
(b) **True** - Every rational number is a real number.
(c) **False** - An irrational number has a non-repeating, non-terminating decimal.
(d) **False** - Natural numbers start from 1, so 0 is not a natural number.
(e) **True** - $\frac{22}{7}$ is a fraction, making it a rational number.

4. **Why is $\sqrt{4}$ rational but $\sqrt{2}$ is not?**

The square root of 4 is ± 2, which are integers and hence a rational number. However, $\sqrt{2}$ is not a perfect square and has a non-repeating, non-terminating decimal representation, making it irrational.